

DOCUMENT RESUME

ED 049 932

SE 011 125

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TITLE A Framework for the Study of Instruction in Mathematics. Science, Technology, and Instruction: A Rationale for an Instructional Research and Development Strategy.
INSTITUTION Pennsylvania State Univ., University Park.; State Univ. of New York, Genesco.
PUB DATE Feb 71
NOTE 44p.; Paper presented at the Annual Meeting of the American Educational Research Association (February 4-7, 1971, New York City, N.Y.)
EDRS PRICE MF-\$0.65 HC-\$3.29
DESCRIPTORS *Curriculum Design, Curriculum Development, *Curriculum Research, Learning Theories, Mathematical Concepts, *Mathematics Instruction, Research, *Research Methodology, *Scientific Methodology

ABSTRACT

Scientific Method rather than trial and error methods should be used to search for instructional algorithms. The Scientific Method is discussed with implications for this purpose. Suggested is the inclusion of theoretical and technological foundations in formulating instructional theory. Conditions needed for adequate instructional research are: satisfying the essential conditions of scientific theory and use comprehensive strategies. Conditions necessary for an adequate instructional system presented are: (1) if instructional problems overlap, construct useful sets of generalizations for solving classes of instructional problems, (2) represent the instructional system as a theory, (3) the system must be capable of replication. An illustration encompassing the research methodology presented is included. Presentation of equivalent fractions through the mode of graphics is discussed. A symbolic language was developed and examples of statements utilizing this language are presented. Using a Gagne' type learning hierarchy, an instructional sequence was designed (although not reported) to achieve selected instructional objectives (also not reported). (JG)

ED049932

A FRAMEWORK FOR THE STUDY OF
INSTRUCTION IN MATHEMATICS

Science, Technology, and Instruction:

A Rationale for an Instructional Research and Development Strategy

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*A Paper Presented at the Annual Meeting of the
American Educational Research Association
New York, February, 1971*

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Preface

The position underlying the work discussed in this paper is that the study of instruction clearly has not been fruitful, and that the failure to make substantial advance in knowledge of instruction is due to three basic reasons:

- (1) The purposes of rational study of instruction have not been set forth with adequate precision and justification;
- (2) The conditions necessary to achievement of adequate purposes have not been set forth in conjunction with precise and systematic justification, nor have they been framed in a structure of sufficient power;
- (3) There have been remarkably few instructional studies of sufficient quality to serve as models for research.

The paper is not easy reading. Its language is not familiar to educationists, not even "educational researchers." It is terse and "tightly packed." Its statements are like icebergs; beneath the surface there are implications to be explicated and examined.

To be comprehended, the paper is likely to take several readings. The statements require careful study and reflection; to pass over them lightly without critical examination is to miss their significance and force.

If the analyses and constructions presented in this paper have validity, then the clear implication is for a substantial change in the kinds of instructional investigations worth undertaking. A significant corollary to the preceding proposition is that the rationale and the studies described herein represent an open and direct challenge to the policy makers governing investments in the study of curriculum and instruction, for the foundations upon which these fields of study and decision stand are in direct contradiction to the conditions of adequate instructional research, and the justifications for them, presented here.

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SCIENCE, TECHNOLOGY, AND INSTRUCTION:
A RATIONALE FOR AN INSTRUCTIONAL RESEARCH AND DEVELOPMENT STRATEGY

by Ralph T. Heimer
and
John J. Lottes

1. INTRODUCTION

1.1. The Problem

In every curriculum construction effort, decisions must be made about how to structure the content, and about how to design and order instructional tasks. Unfortunately, at the present time neither adequately justified rules nor precise, empirically testable sets of hypotheses are available for guiding such instructional decisions. It follows, by force of circumstance, that instructional actions are not governed by deliberate, systematic thought; on the contrary, instructional moves characteristically are based on imprecise, inadequately formulated "practitioner's maxims" or "hunches"--for which there is neither adequate logical nor adequate empirical support. Consequently, those people who are responsible for instruction at the level of practice not only employ a trial and error paradigm for making instructional decisions, but do so under circumstances which virtually prohibit any substantial opportunity for programmatic improvement (viz. to learn from their mistakes).

Rational analysis of the nature of instruction leaves little doubt

as to the ultimate goal of the study of instruction. The aim is to learn how to construct adequate instructional algorithms. The term "algorithm" as it is used here is consistent with the informal definition provided by Hull [5] in which he stated that:

An algorithm is a procedure for solving a problem. Usually one has in mind a class of problems, and the algorithm is a procedure which can be applied to any member of the class, not to just one particular problem. An important requirement is that the individual steps in an algorithm must be completely unambiguous. Moreover, the algorithm must always produce the solution to the problem and it must do so in a finite number of steps.

Thus, every instructional algorithm would consist of a description of procedures for constructing instructional sequences that, upon being executed, will maximize the likelihood of the attainment of a given class of objectives under some specified set of learner circumstances. Instructional algorithms, therefore, would deal with empirically validated "cause-effect" relationships between instructional actions and learning outcomes. Their availability would provide educators with the ability to describe, to explain, and to predict effectively within the instructional domain.

If it is the *raison d'être* of the study of instruction to acquire the foregoing abilities, then not only is it appropriate to seek the knowledge necessary to construct adequate teaching algorithms, but it should also be recognized that the study of instruction is appropriately viewed as an experimental science and may be treated as such. Indeed, it is the purpose

of the remainder of this paper to consider how the scientific model might be employed as a template for developing a science of instruction, and to briefly describe an instructional research strategy constructed under the requirements of such a paradigm.

1.2 The Nature of the Instructional Claim and its Implications

Associated with every rational instructional program are claims (or assumptions or expectations) of the form:

Under the set of circumstances C , the set of instructional activities I will cause student S to attain a set of objectives O ;

in most instances, however, the knowledge of circumstances is inadequate, the objectives are vague, and the choice of instructional actions is not made at a deliberate level of awareness.

In any event, if an instructional system is operating subject to the conditions of an instructional claim of the type cited above, there should be some rational basis for deciding upon the specific instructional actions to be taken at any given time. In the absence of an adequate knowledge structure for guiding ones decisions, most instructional specialists (and hence curriculum development projects) have resorted essentially to what will be referred to here as a trial and error paradigm for making instructional decisions. Presumably such a procedure can, given sufficient time,

lead to a reasonably satisfactory curriculum product--in the sense that some reasonably high proportion of students from a specified target population could be expected to achieve a reasonably high proportion of the objectives encompassed by the curriculum. Unfortunately, there can be no assurance that a curriculum product developed under the conditions of the trial and error paradigm is efficient. Of much greater concern, however, is the fact that the efficiency of the paradigm itself--by definition--is not subject to progressive improvement.

It is argued here, therefore, that the trial and error strategy is inadequate as a vehicle for the study of instruction. Rather, an attempt should be made to build the kind of knowledge base needed to predictably relate instructional actions to learning outcomes. It is believed that such a knowledge base must consist of rules for action that are minutely explicit, and that provide for optimal information flow between instructor and student, and vice versa. In short, the aforementioned rules would constitute the mortar needed to build instructional algorithms, and hence instructional systems, that meet minimal conditions of adequacy. These conditions are discussed below.

2. SCIENCE AS A MODEL FOR THE STUDY OF INSTRUCTION: EXPLICATION AND JUSTIFICATION

The assumption has been made that a rational study of instruction constitutes a necessary step in advancing the power to explain, predict,

or control empirical events relevant to instructional claims.

In an attempt to maximize the likelihood and efficiency of acquisition of the power of explanation, prediction, and control, the scientific model will be invoked as a template for the study of instruction. To invoke the scientific model as a template for the study of instruction means to: (1) use the characteristics of scientific theory as a model for the characteristics of instructional systems; (2) use the ways of evaluating scientific theory as models for the ways of evaluating instructional systems; and (3) use the way of advancing scientific knowledge as a model for the way of advancing knowledge in the instructional domain.

One task of this section is to set forth the fundamental characteristics of the scientific model. A second task is to justify the use of the scientific model as a template for the study of instruction. Finally, essential characteristics of the scientific model will be taken into account in specifying the conditions of an adequate instructional research and development strategy.

2.1 Fundamental Characteristics of the Scientific Model

What are the essential characteristics of scientific theory? Carl Hempel, in *Aspects of Scientific Explanation* [4], has described the structural characteristics and the functional characteristics of scientific theory.

The structure of scientific theory satisfies the requirements of a deductive system. The vocabulary of a theory (*i.e.* the extralogical terms; the constructs which have empirical referents--at least indirectly) consists

of undefined terms (*i.e.* primitive terms) and defined terms. The set of sentences of a theory consists of axioms (primitive sentences) and theorems (derived sentences). In the theoretical structures of empirical science, the deduced theorems are typically called hypotheses. The structure is formulated under explicit rules for forming sentences and for judging the validity of a deductive inference.

On the other hand, the functional characteristics of a theory of empirical science are defined by the power of the theory to predict and explain empirical phenomena. Explanatory and predictive power is made possible by the deductive rules governing the theory in conjunction with operational definitions which give the extra-logical terms empirical import. These are the means for bridging the gap between the abstract statements constituting a theory and the concrete observational objects and events to which the theory is relevant.

To claim to have formulated a theory is not sufficient. The theory formulated must be examined and judged. Adequate evaluation of scientific theory must include examinations relevant both the structural characteristics and the functional characteristics of the theory.

What are the ways of evaluating scientific theory? Karl Popper, in *The Logic of Scientific Discovery* [11], has identified four different ways of evaluating scientific theory: (1) assessing the coherence of the theory by making logical comparisons among its statements to determine if there are inconsistencies; (2) assessing the logical form of the theory to

determine if it has the characteristics of a theory of empirical science; (3) assessing the potential of the theory for contributing to scientific advance if it should withstand all logical and empirical tests; comparison with other theories is the methodology for this assessment; and (4) assessing the correspondence of the theory with real world objects and events; the methodology for this assessment requires both deduction of more concrete statements from the abstract theory statements, and inductive inference based on empirical observations which result in judgments of support or non-support for the truth of the theory statements.

The essential characteristics of scientific theory have been identified. The essential characteristics of theory evaluation have been identified. The essential characteristics of the strategies for building fruitful theory now must be identified.

What are the ways of advancing scientific knowledge? Advancement of scientific knowledge involves concurrent efforts to attain ever higher levels with respect to extralogical terms, the linguistic structures which house the extralogical terms (e.g. the theoretical constructs, or abstract concepts), and the scope and validity of those structures. The essence of the way in which this complex bootstrapping process is accomplished is described in the following paragraphs.

The ways of scientific inquiry--retroduction, deduction and induction--have been explicated and illustrated by Elizabeth Maccia in two papers: *Ways of Inquiry* [10] and *The Model in Theorizing and Research* [9]. Retroduction is the creative process of formulating theory statements. If a theory state-

ment (a hypothesis) has been set forth, it must be tested. Deduction and induction are the ways of inquiry available for distinguishing adequate statements from inadequate statements.

Deductive inference is invoked to explicate hypotheses in terms of lower level consequents. These consequents must be of such concreteness and specificity that between-observer and within-observer invariance can be achieved in reporting relevant observational phenomena.

Inductive inference is invoked to bridge the directional gap from the reports of observational phenomena to the derived consequent statements, and ultimately to the hypothesis under test. The inductive inference is represented in the form of a judgment as to whether a hypothesis has been tentatively confirmed or disconfirmed.

The strategy for scientific advance consists of a continuing series of repeating retroduction-deduction-induction cycles. Either directly or indirectly they are invoked over and over again on every aspect, and at every state, of the knowledge structures under development: the extra-logical vocabulary, the statements of the structures, and the relations among the structures. No assertion is free from critical examination and judgment except the axioms of the logical meta-system under which all scientific inquiry operates.

The consequence of the series of retroduction-deduction-induction cycles is a non-terminating set of theoretical structures characterized by increasing predictive and explanatory power. Inadequate structures

are identified and abandoned, or are replaced by modified structures which in turn become objects of evaluation. Strong constructions can withstand the most rigorous tests; investments are made in these strong constructions.

2.2 Science as a Model for the Study of Instruction

The intent of this section is to justify the claim that the essential characteristics of empirical science constitute an adequate model for the study of instruction. Satisfactory justification of the claim requires execution of the following steps:

- (1) Identification of the purposes of the study of instruction.
- (2) Identification of the essential characteristics of empirical science.
- (3) Specification of the conditions under which the characteristics of one field of study constitute an adequate model for another field of study.
- (4) Demonstration that the essential characteristics of empirical science constitute an adequate model for the study of instruction.

The first two steps already have been executed. That is, (1) the purposes of the study of instruction have been identified, and (2) the essential characteristics of empirical science have been identified.

The last two steps now will be executed.

2.2.1 Conditions of an Adequate Model for a Field of Study

Assume that one field of study, *A*, is used as a model for a second field of study, *B*. Then under what conditions should *A* be judged an adequate model for *B*--where this judgment must be made prior to utilization of the essential characteristics of *A* in the field of study, *B*? This judgment clearly must be made on logical grounds, for all possible empirical tests would necessarily be conducted subsequent to invoking the model.

It seems reasonable to assume that a scholar in one field would not wish to borrow from the knowledge structures of a second field if the second field were not distinguished by quite noteworthy knowledge structures. That is, one would not wish to borrow from a field where little advance has occurred; one would not want to bet on an "also-ran." These considerations suggest that clearly demonstrable progress should be taken into account as a condition of *A*, where field of study *A* is judged an adequate model for field of study *B*.

Even if one field of study, *A*, has demonstrated progress of considerable magnitude, it seems that one would not wish to use *A* as a model for a second field of study, *B*, if the purposes of *A* and *B* were incompatible, or if it could not be shown that *A* and *B* were analogous fields of study in certain significant respects. That is, correspondence between *A* and *B* also should be taken into account in specifying the conditions of an

adequate model.

The foregoing considerations lead to the following conditions of an adequate model for a field of study:

Field of study, A , is an adequate model for field of study, B , if and only if:

1. A is characterized by some knowledge structures which have been validated by extensive logical and empirical tests, and for which the claim of generative power can be reasonably supported.
2. If P_A = set of purposes associated with A , and P_B = set of purposes associated with B , then
 - (a) P_B is a subset of P_A
 - or (b) $P_B - P_A$ is small relative to P_B
 - or (c) P_A is identical to P_B

2.2.2 Justification of Science as an Adequate Model for the Study of Instruction

The first three steps of the procedure for justifying the claim that empirical science is an adequate model for the study of instruction have been executed. One step remains; it must be demonstrated that the relation between the essential characteristics of empirical science and the study of instruction satisfies the defining conditions of the relation: "Field of study A is an adequate model for field of study B ."

The first condition of the adequate model relation is clearly satisfied by at least the natural sciences, if not the behavioral sciences. That is, substantially validated theoretical structures characterize the natural sciences. One might point to the knowledge structures of nuclear physics or the knowledge structures utilized in achieving the objective of safely transporting men on a round trip between earth and the moon. These instances should suffice.

The second condition necessary to support the claim that one field of study is an adequate model for a second field of study specifies a non-empty intersection between the two sets of purposes. The purposes of the study of instruction as set forth in this paper are explanation, prediction, and control; explanation and prediction were also specified as the functional characteristics of scientific theory. Hence, the second condition of an adequate model is satisfied by the relation between the purposes of empirical science and the purposes of the study of instruction.

Since all requirements of the definition of the adequate model relation have been satisfied, it is concluded that the essential characteristics of empirical science constitute an adequate model for the study of instruction.

It is of value also to examine the relationship between the instructional claim (see page 2) and the hypotheses of empirical science. Both claims are of the form: If X then Y . Further, in the claims of both empirical science and instructional study, the statements replacing X and Y must have empirical import. That is, to satisfy the specified purposes of study,

the statements must be interpretable against real world phenomena.

Furthermore, if the study is rational--whether in empirical science or instruction--accountability must be established, and the nature of the claims in both cases demands empirical test.

One last examination is enlightening. In *The Logic of Scientific Discovery*, Karl Popper has examined a variety of potential ways of distinguishing fields of study which are science from fields of study which are not science. Ultimately he proposes a single criterion for distinguishing science from non-science. The demarcation criterion is: If the claims of the field of study are capable of being falsified on the basis of empirical evidence, then the field of study is appropriately labeled "science." Therefore the study of instruction belongs, by definition, to the extension of the concept 'empirical science'; consequently, it is appropriate to employ the methods of science in instructional investigations.

Two lines of reasoning were followed in justifying the claim that empirical science is an adequate model for the study of instruction: (1) empirical science was examined to determine if it satisfies the conditions of an adequate model for the study of instruction, and (2) the nature of an instructional claim was compared against the demarcation criterion for distinguishing science from non-science. Both lines of reasoning yielded the same conclusion: the scientific model is appropriately applied to the study of instruction.

3. WAYS OF ADVANCE IN INSTRUCTIONAL PRACTICE

The purposes of rational study of instruction have been set forth. The scientific model has been proposed and justified as a template to guide the study of instruction. If this line of reasoning is valid, then the fundamental characteristics of the scientific model constitute conditions necessary to the fruitful study of instruction. However, the characteristics of the scientific model do not constitute sufficient conditions for productive and continuing advance in the study of instruction. In order to take into account conditions not yet considered, the scope of the examination must be extended; this is the purpose of the present section.

3.1 Relation Between Instructional Practice and the Study of Instructional Practice

To provide the framework for a broader analysis, 'instruction' will be viewed as a field of practical action. Under this view the ways of advance will be identified and explicated, and their import will be made clear.

In *Praxiological Sentences and How They Are Proved* [8], Taddeus Kotarbinski has set forth the aims and ways of inquiry in a field of practical action. The purpose of study of practical actions is to develop practical directives--the theorems of the field--which specify relations between particular courses of action and their consequences in terms of achievement of specified ends. Fruitful study of a field of practical action will result in directives, or theorems, which specify actions of increasing efficiency relative to specified ends. In highly simplified form, the theorems are cast in the form:

"In the presence of circumstances C , course of action A will cause (with some specified probability) the achievement of objective θ ." In view of the aims of the study of practical action and the empirical nature of the theorems developed, the study of practical action is appropriately scientific. As a matter of fact, Kotarbinski defines 'praxiology' as the "science of efficient action."

Under Kotarbinski's clarifying analysis, the demarcation between practical action and the study of practical action is made precise. The study of practical action is science, and its purpose is to develop knowledge structures which have the power to predict, explain, and control some specified domain of empirical phenomena; that domain consists of practical actions. Practical actions are the phenomena to be predicted, explained, or controlled; these phenomena do not belong to the extension of the concept 'science.'

The relation between the study of instruction (a science) and instructional actions (the phenomena under study) is identical with the relation explicated in the preceding paragraph.

It should also be noted that the previously discussed "instructional claims" are essentially the same as Kotarbinski's "praxiological theorems." Both are algorithmic in nature and specify the conditions under which a specified course of action is likely to achieve the intended ends. It is clear that the efficiency associated with a field of practical action is dependent upon the power and validity of the theorems developed through scientific study of relevant actions. That is, efficiency of practical action is a function of the knowledge structures built through the study of the field of practical action.

3.2 Conditions of Advance

Efficiency and power of instructional practice is a function of the knowledge structures generated by the study of instructional practice. What are the conditions under which the scientific study of instruction is most likely to result in substantial gain?

In his discussion of praxiological theorems, Kotarbinski also analyzed the circumstances under which substantial advances in a field of practical action are most likely to occur. Kotarbinski's analysis yielded the conclusion that advances are most likely to occur in the presence of some combination of these conditions:

- (1) There is an advance in the theoretical foundations of potential relevance to the field of practical action.
- (2) There is an advance in the technological foundations of potential relevance to the field of practical action.
- (3) Available, but previously ignored, information from potentially relevant theoretical or technological foundations is utilized by the field of practical action.
- (4) There is a different selection or different ordering of actions in the field of practical action.

It is enlightening to examine further Kotarbinski's analysis, where instruction is the field of practice under consideration. In this event,

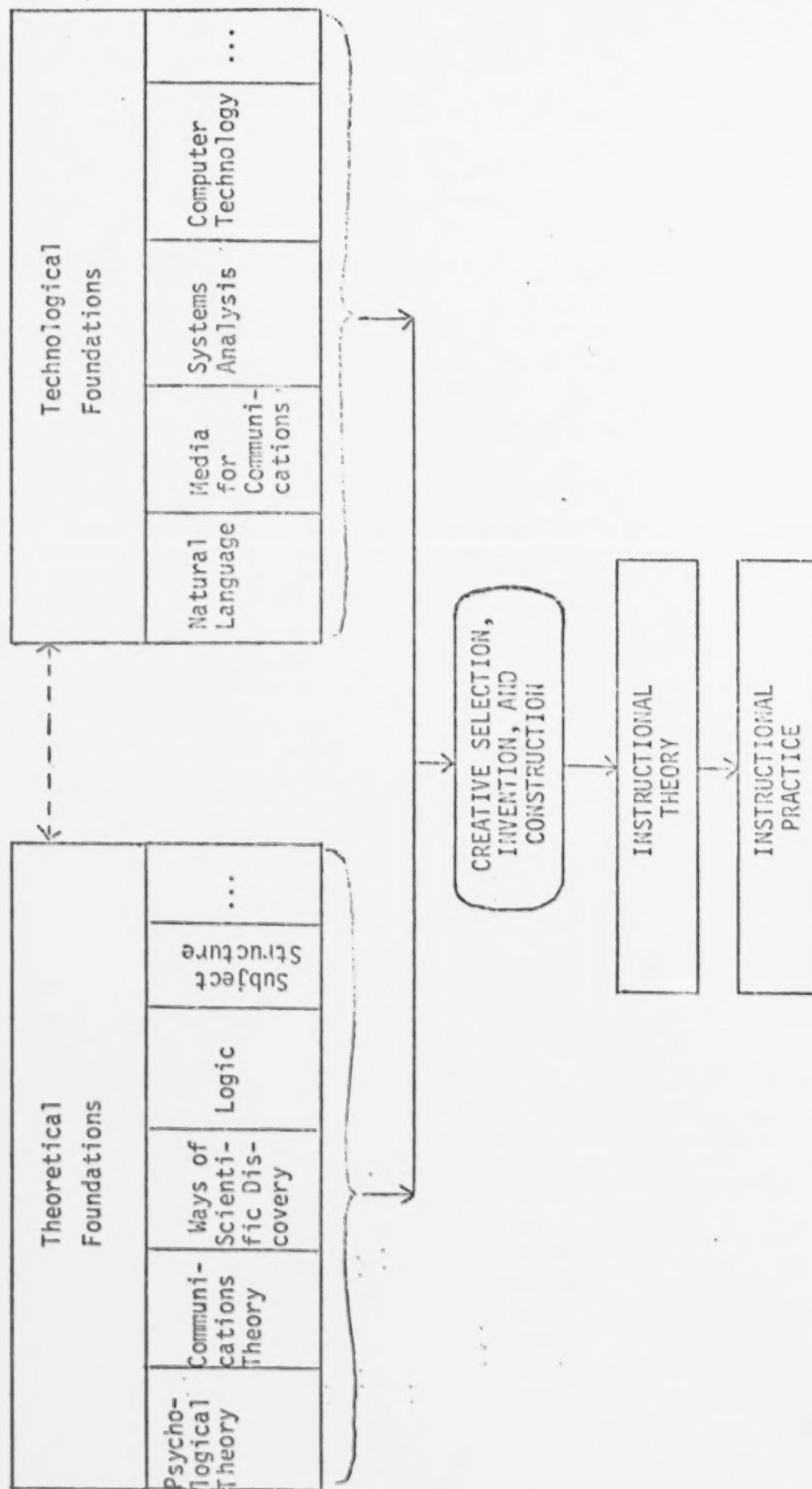
some potentially relevant theoretical foundations are learning theory, theory of action, communication theory, logic, the knowledge structures of the disciplines under instruction, and so on. Some potentially relevant technological foundations are available languages, electronic media, computer technology, systems engineering, and so on.

The relations of the theoretical foundations and technological foundations to the sets of statements which guide instructional practice are displayed in the graphic representation on the following page.

A Kotarbinskian view of the relation of the theoretical foundations and technological foundations to instructional theory statements and instructional practice distinguishes clearly between the tasks of the disciplines--humanities, formal sciences, natural sciences, behavioral sciences--the tasks of the technologist, and the tasks of the educational investigator.

In the study of instruction, information from the theoretical and technological foundations is examined and judged as to its potential for contributing to the knowledge structures which are unique to the instructional domain. Information which is judged to be capable of being accommodated by a knowledge structure of instructional science and which also has reasonable potential for contributing substantially to the scientific worth of the knowledge structure may be selected by the educational investigator for utilization in some way. The information examined, the information selected, and the way in which selected information is manipulated and utilized is the free choice of the educational investigator.

RELATION OF THEORETICAL AND TECHNOLOGICAL FOUNDATIONS
TO INSTRUCTIONAL THEORY STATEMENTS
AND INSTRUCTIONAL PRACTICE



His only burden, under the scientific model, is for rigorous testing of his unique structures under the complete range of evaluative methods characteristic of empirical science.

It is enlightening to examine the complementary roles played by the theoretical and technological foundations of potential relevance to a field of practice. The theoretical foundations provide variables, relations, and forms which may be taken into account in the theorems associated with a field of practice. For example, in the studies described later, the reversibility relation of Piaget's theory of cognitive development was utilized in building theorems of instructional science.

While the theoretical foundations provide potentially relevant substance or form for knowledge structures of sciences of efficient action, the power to manipulate a particular set of variables or to take a particular set of relations into account is a function of the available technological foundations. For example, the instructional studies to be described in section 5 involve deliberate within-task variation of modes of representation in conjunction with complex branching options which are contingent upon individual pupil responses. The systems of hypotheses--major and auxiliary--discussed in this paper were formulated in the presence of computer technology as a necessary condition of application. In the absence of computer technology these systems of hypotheses would, for the time being, be of little significance to the study of instruction since they would not be capable of empirical test; hence they would have no application to instructional practice.

4. CONDITIONS OF ADEQUATE INSTRUCTIONAL RESEARCH AND DEVELOPMENT

The ways of advance in the study of instruction have been identified and explicated. The scientific model has been examined and justified as a template for such a study. The conclusions reached by these lines of reasoning imply that certain requirements must be satisfied if the study of instruction is to be potentially fruitful. These requirements are set forth in the following sections as "conditions of adequacy."

4.1 Conditions of an Adequate Instructional Research Strategy

The following set of statements specifies the conditions of an adequate strategy for instructional research:

- (1) Development of the extralogical vocabulary of instructional theorems should take into account knowledge of the potentially relevant theoretical and technological foundations. This requirement demands continuing research into these foundations, as well as the philosophical foundations.
- (2) If individualization of instruction is desired, then the knowledge structures formulated should take the capability of computer technology fully into account.
- (3) Instructional inquiry should satisfy the essential conditions of the scientific model:
 - a) the knowledge structures of instructional studies should satisfy the structural and

- functional characteristics of scientific theory;
 - b) the knowledge structures formulated should be judged under each of the four ways of evaluating scientific structures;
 - c) the research strategy should invoke the continuing retroduction-deduction-induction cycles of the scientific paradigm.
- (4) Alternating philosophical and empirical inquiries should be deliberately undertaken to develop the extralogical language to more refined levels.
- (5) Continuing study should be undertaken into the full range of scientific curriculum inquiry and the relations between the different logical levels of curricular and instructional knowledge structures; that is, the strategy should be a comprehensive strategy.

4.2 Conditions of an Adequate Instructional System

If each instructional problem (how to cause students to attain objective θ) is unique, then there can be no useful general procedures; under such circumstances each instructional problem must be solved by trial and error methods. However, if substantial overlap exists among different instructional problems, then it might be possible to construct useful sets of

generalizations for solving classes of instructional problems. Of course, to invoke any set of generalizations to solve a problem, two conditions must be satisfied:

- (1) The set of generalizations must exist.
- (2) The problem must belong to a class of problems for which the set of generalizations provides a guaranteed method of solution.

These two conditions must be taken into account in establishing criteria for an adequate instructional system.

Another fact which needs to be taken into account is that the worth of an instructional system, I , is dependent upon the validity of the claim:

If I is invoked in the presence of circumstances, C ,
then the student, S , will attain a set of objectives, θ .

Such a claim demands an empirical test. Any meaningful empirical test requires the reproducibility of results over different occasions; a necessary condition to reproducible results is the ability to manipulate a set of independent variables in the same way over different occasions.

Every instructional system can be represented in a variety of formats and at different levels of abstraction. Distinctions will be made here among three different levels of representation: theory level, design level, and interactive level. Descriptions of the use of these terms follow.

Theory level:* The highest level of abstraction; the mode of representation is symbolic. At the theory level, an instructional system is represented as a set of statements formulated in mathematics terms. Precision, coherence, and generative power are conditions necessary to a theory level representation of non-trivial import.

Design level:* The mode of representation is symbolic or iconic. At the design level, an instructional system is represented as a set of sequential tasks, particular branching instructions, particular evaluative procedures, data storage procedures, feedback messages, particular flowcharts, and the like.

Interactive level: At the interactive level, the design is implemented. A pupil exchanges information with the instructional system. The interactive level of representation occurs when the act of influencing a pupil occurs.

These considerations lead to a third condition which must be taken into account in establishing criteria for an adequate instructional system:

*It should be noted that theory and design are merely two different levels in a single hierarchical system of statements. The point of division between the theory level and design level descriptions is an arbitrary matter.

- (3). The instructional system must be capable of replication over occasions at the interactive level.

The foregoing reflections, in toto, lead to the following conditions of an adequate instructional system:

- (1) The instructional system must be represented at the theory level, preferably as a set of well-formulated mathematical statements. This system of statements must satisfy logical tests of precision, coherence, and potential generative power.
- (2) The instructional system must include an associated class of instructional objectives over which the system guarantees some specified level of efficiency. These objectives must take into account stimulus conditions, response conditions, and student circumstances.
- (3) The instructional system must yield design-level and interactive-level consequents which are capable of replication.

5. AN ILLUSTRATION OF A STRATEGY FOR THE DEVELOPMENT OF KNOWLEDGE ABOUT INSTRUCTION IN MATHEMATICS

The purposes of rational study of instruction have been identified. The scientific model and the ways of advance in a field of practical action

have been proposed and justified as sources of information of potential worth to the study of instruction. The conclusions of these examinations have been set forth as conditions of an adequate instructional research and development strategy.

A research project is now underway at The Pennsylvania State University in which a deliberate effort is being made to study selected instructional phenomena under the conditions of adequacy which have been set forth. Specifically, an attempt is being made to construct a framework of investigation that will promote the long-term development of the type of knowledge base needed to predictably relate instructional actions to learning outcomes. It is the purpose of this section to describe the essential characteristics of the line of research that has been undertaken.

5.1 The Modes of Representation Studies

Two studies have been completed, several are now in progress, and numerous others are planned all of which deal with modes of representation (MR) variables. The MR constructions employed in all investigations to-date are consistent with those stated by Bruner [1]:

Any domain of knowledge (or any problem within that domain of knowledge) can be repeated in three ways (a) by a set of actions appropriate for achieving a certain result (enactive representation), (b) by a set of summary images or graphics that stand for a concept without defining it fully (iconic representation), and (c) by a set of symbolic or logical propositions drawn from a symbolic system that is governed by rules or laws for forming and transforming propositions (symbolic representation).

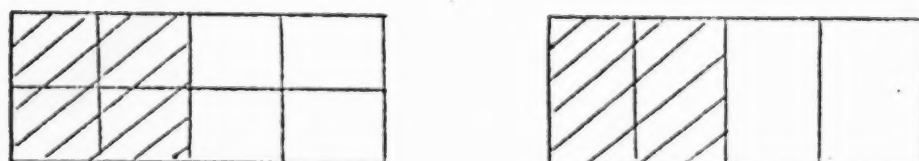
5.1.1 The Impact of MR variables on the Formulation of Objectives

Interestingly, the interaction of the study of MR variables and the task of designing a framework for formulating instructional objectives (another requirement of an adequate instructional system--see p.24) immediately led to the idea of classifying objectives as MR ordered pairs. For example, consider the objective:

<u>Given</u>	<u>Required Performance</u>	<u>Criteria</u>
Two partially shaded rectangles which depict a pair of equivalent fractions.	Write the pair of equivalent fractions suggested by the diagrams.	75% of 4 items in time t .

The above objective defines an unambiguous "test" pool, as do all of the objectives that have been written. An instance in the present case would be:

Write the pair of equivalent fractions suggested by the following diagrams. _____



The foregoing objective would be classified as an (I, S) ordered pair since the mode of representation of the given (the input) is iconic, and the mode of representation of the required performance (the output) is symbolic.

The scheme of classifying objectives in the way described above then gave rise to the notion of constructing an MR matrix of objectives for any given unit of content. The collection of objectives arrived at in this way

is called a cluster and an illustration of one is provided in Table 1 below.

Table 1

A Cluster of Objectives for the Concept of Equivalent Fractions

O U T P U T M O D E	S	<u>Given:</u> A pair of equivalent fractions depicted with Cuisenaire rods. <u>Req. Perf.</u>¹ Write a pair of equivalent fractions suggested by the rods.	<u>Given:</u> Two partially shaded rectangles depicting a pair of equivalent fractions. <u>Req. Perf.</u> Write a pair of equivalent fractions suggested by the diagrams.	<u>Given:</u> A fraction. <u>Req. Perf.</u> Select from a set of 4 fractions the one that is equivalent to the given fraction.
	I	<u>Given:</u> A pair of equivalent fractions depicted with Cuisenaire rods. <u>Req. Perf.</u> Select from a set of 4 diagrams the one which depicts the same equivalence.	<u>Given:</u> Two partially shaded rectangles depicting a pair of equivalent fractions. <u>Req. Perf.</u> Select from a set of 4 diagrams the one which depicts the same equivalence.	<u>Given:</u> A fraction. <u>Req. Perf.</u> Select from a set of 4 diagrams the one which depicts an equivalent fraction.
	E	<u>Given:</u> A pair of equivalent fractions depicted with rods. <u>Req. Perf.</u> Construct the same equivalence using different colored rods.	<u>Given:</u> A Pair of partially shaded rectangles depicting two equivalent fractions. <u>Req. Perf.</u> Demonstrate the equivalence of the fractions suggested using Cuisenaire rods.	<u>Given:</u> A pair of equivalent fractions. <u>Req. Perf.</u> Demonstrate the equivalence using Cuisenaire rods.
		E	I	S
		INPUT MODE		

¹Required Performance

The conceptualization of the IR matrix idea and the resultant clusters of objectives provided by them has several important attributes. First, it has made possible the systematic construction of worthy instructional objectives some of which apparently have been overlooked by instructional specialists and curriculum developers. (This assertion is based on the fact that a preliminary search of selected school mathematics curricular materials revealed no attempt to provide instructional experiences related to certain identified objectives.) If this assertion is correct, it is an outcome of great importance, for it most assuredly will lead to the development of instructional actions here-to-fore never attempted. Second, it presents a highly tractable opportunity to investigate an age-old problem in pedagogy--namely the proper presentation order of instructional experiences with respect to modes of representation. (It is believed here that most extant "knowledge" about this matter is best categorized as speculation.) In any event, the problems of learning 1) how to attend to the objectives in a cluster, and 2) the interrelationships among them (e.g. how the acquisition of one objective in a cluster affects the acquisition of another) has opened for scientific study a huge domain of important instructional phenomena.

5.1.2 The Construction of a Language for Representing the Modes of Representation--Objectives System

In an attempt to cope with the foregoing problems--and to do so by adhering to the requirements imposed by applying the scientific model, the

first step that was taken consisted of trying to develop a theory-level representation of the system. This necessitated the construction of a symbolic language which it was hoped would afford an opportunity to draw upon the underlying logical meta-system, and hence increase the generative power of the representation. The basic elements of the MR language that was constructed is summarized below.

- $\hat{O}$: A cluster of objectives, $\hat{O}$, is defined to be a set of objectives all of which pertain to the same mathematical content, but which differ in their input-output modes of representation.
- o_x : An arbitrary objective from some cluster $\hat{O}$.
- E : The enactive mode of representation; it is considered to be synonymous with the object mode and is such that the physical characteristics of the exemplar can be felt and manipulated.
- I : The iconic mode of representation; it is synonymous with the picture mode, and is such that the physical characteristics or qualities can be viewed, but cannot be felt or manipulated independent of the medium in which it is presented.
- S : The symbolic mode of representation; it is taken to be a form of words or symbols (usually mathematical) having only ideational relation to the referent.

$\dot{M}$: The set consisting of the modes of representation.

$$M = \{E, I, S\}$$

$C(\theta_k)$: The classification of the objective θ_k as an MR ordered pair (N_i, N_o) where N_i is the mode of representation of the input phase of θ_k and N_o is the mode of representation of the output phase.

$N_i \rightarrow N_o$: An instructional sequence whose purpose is the accomplishment of an objective θ_k .

$\overline{A} \atop N_i \rightarrow N_o$: An instructional sequence deemed adequate according to some well-formulated criterion; for example, let n be the number of the students who fail to reach criterion on a pre-test for an objective θ_i , let s be the number of students reaching criterion after instruction, and let t be the greatest integer less than $0.8n + 0.5$. Then the instructional sequence for θ_i is deemed adequate if and only if $s \geq t$.

$\overline{A}(N_i, N_o)$: The achievement of an instructional objective without explicit instruction.

5.1.3 The Construction of Statements With the MR-Objectives System and their Potential Significance

By using the above symbolism, the original questions of interest can be expressed succinctly--as illustrated by the following example.

$$\overline{A} \\ (I \rightarrow S) \Rightarrow \overline{A}(S, I)$$

This statement may be interpreted to mean that if explicit instruction to criterion is given on an objective with iconic input and symbolic output, then without explicit instruction, achievement of an objective of like content with symbolic input and iconic output will occur. Altogether, some 72 conditional statements of the above type can be formed from the 3x3 MR matrix of objectives, and each can be subjected to empirical test. Moreover, the outcomes could have significant implications for designing instructional algorithms; for example, suppose that the statement

$$\overline{A} \\ (I \rightarrow S) \Rightarrow \overline{A}(S, I)$$

receives strong empirical support over some specified class of objectives, but that the statement

$$\overline{A} \\ (S \rightarrow I) \Rightarrow \overline{A}(I, S)$$

does not. In such a situation; if it is desired that both the (S, I) and (I, S) objectives be achieved, the presentation order would call for the $I \rightarrow S$ sequence first--in the interest of instructional efficiency. Actually, some of the outcomes of the research conducted to-date hint at the possibility of outcomes of this sort.

There are a multitude of additional statements about MR clusters that can be constructed and tested; fortunately, most, if not all, of these are readily obtainable because of the logical form of the language that has been developed. In addition to the simple conditionals described above, the major forms of such statements may be classified as follows:

1. Equivalence Statements;
2. Transitivity Statements;
3. Conjunctive Statements

A little reflection on the above possibilities reveals that they are not only numerous, but that they have considerable meaning and significance. This seems to be adequate evidence in support of the claim that the language construction has made a substantial contribution to the generative power of the system.

5.2 The Utilization of Information From a Relevant Theoretical Structure to Enhance Knowledge About the MR-Objectives System

As was pointed out earlier, continued improvement of a field of practical action (*e.g.* instruction) is contingent in part upon the advance

of knowledge in potentially relevant theoretical foundations, or upon the application of available information not previously utilized. An advance of highest significance has occurred in the Penn State project, one that has resulted from the utilization of information from the cognitive theory of Piaget--information which had not previously been taken into account. This instance, described briefly below, provides support of compelling force for the conditions under which the strategy for the Penn State project has been developed.

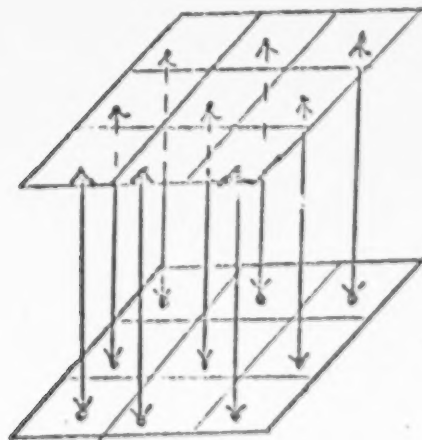
One of the instructional objectives involved in several of the project studies was as follows:

<u>Given</u>	<u>Required Performance</u>	<u>Criterion</u>
A set of Cuisenaire rods, one of which has been designated as the unit, and a proper fraction $\frac{x}{y}$ for which there exists a rod whose length is $\frac{x}{y}$ times the length of the unit rod.	Select from a set of four colored rods the one which represents $\frac{x}{y}$.	2 of 3 items in time t .

An instructional sequence was designed to enable a selected class of learners to achieve this objective--the design being based primarily on a Gagné type

learning hierarchy constructed and approved by several members of the staff. Unfortunately, however, the instructional sequence that was constructed proved to be inadequate (under the criterion for adequacy specified on the bottom of p. 30), and an analysis of it was undertaken in an effort to determine the cause of failure. Since the micro hypotheses (e.g. rule-example patterns, feedback rules, etc.) employed in building the sequence were identical to those used in building other sequences that were judged adequate, two lines of investigation began to emerge: 1) flaws in the learning hierarchy were sought, and 2) the set of micro hypotheses were examined with the thought that they might be insufficiently complete or precise to account for the outcomes being observed.

In the meantime, a breakthrough of major importance occurred when a graduate student who was concerned with Piaget's concept of operational reversibility incorporated this idea with the MR matrix, and discovered that under certain conditions it is possible to create another 3x3 cluster of objectives that is related to the MR cluster, cell for cell, as shown below.



In the case of the instructional situation under study, it was possible to construct a cluster of the foregoing type, and the result of doing so was the development of a set of related objectives which were deemed extremely worthy, but which had been overlooked by the entire staff. (More interesting, however, is the fact that some of these particular objectives apparently have been missed by other curriculum developers as well--in view of the fact that the staff has been unable to find planned instructional experiences related to them in any curricular materials examined to-date.) This outcome, in turn, prompted the development of an entirely different type of instructional sequence--one which is believed to be unusual, if not unique.

The moral of the story is that the conditions under which the Penn State instructional research and development strategy was developed are proving to be fruitful conditions. The specified conditions of an adequate research and development strategy are "proving their mettle." In the case of the above example, the advance in information about the matter of practical concern, *i.e.* the design of an instructional sequence, came about as the result of a deliberate attempt to utilize information from a potentially relevant knowledge structure of the theoretical foundations, namely Piaget's work. (In actual fact, one of the initial studies that was conducted dealt with state reversibilities, *e.g.* (E, S) , (S, E) objectives, and an examination of the reversibility issue in a broader perspective naturally led to the consideration of Piaget's efforts and their implications for our work.) It should be pointed out, however, that a number of other conditions were "right" for the events reported above: 1) the extra-logical

language employed by Piaget in his studies had to be compatible with our extra-logical language, and it was; and 2) our conceptual and linguistic structures had to be characterized by a form and substance capable of accommodating the new information, and they were. It also should be pointed out that our research strategy of insisting upon a conceptual framework which guarantees a connectedness or interrelatedness among studies, but which follows a rational course of action toward an ever-increasing ability to explain, predict and control provided the impetus for the search for relevant "outside" information in the first place.

6. SUMMARY AND CONCLUSIONS

The purposes of rational study of instruction have been identified and the ways of inquiry judged most likely to attain those purposes have been proposed and justified. Essential characteristics of the ways of inquiry have been set forth as "conditions of adequate instructional research." These conditions have been derived from the scientific model in conjunction with the philosophy of practice, and are judged necessary to fruitful instructional research.

Unfortunately, the foregoing conditions have seldom, if ever, been satisfied by a strategy for the study of instruction. If this assertion is valid, then the absence of fruitful instructional theory and adequate instructional algorithms is explained. The conditions necessary to advance in instructional knowledge have not been present.

A program of instructional research developed under the specified "conditions of adequacy" has been described and the significance of component studies demonstrated. Although it would be premature to claim success in the development of fruitful instructional algorithms with reference to mathematics, the theoretical and empirical outcomes to this point are extremely encouraging.

Bibliography

1. Bruner, Jerome S. *Toward a Theory of Instruction*. Cambridge, Mass.: Harvard University Press, 1966.
2. Farris, Dan C. *Toward a Theory of Sequencing: Study 1-2: An Exploration of Selected Relationships Among the Enactive, Iconic, and Symbolic Modes of Representation*. (Unpublished doctoral dissertation, The Pennsylvania State University, 1970.)
3. Heimer, Ralph T. "Conditions of Learning in Mathematics: Sequence Theory Development." *Review of Educational Research*, Vol. 39, No. 4, October, 1969.
4. Hempel, Carl G. *Aspects of Scientific Explanation*. New York: The Free Press (MacMillan Co.), 1965.
5. Hull, T.E. *Introduction to Computing*. Englewood Cliffs, N.J.: Prentice Hall, 1966.
6. Jorgenson, Christabel *Development of an Adequate Instructional System With Reference to Discovery Learning*. (Unpublished masters thesis, Texas Christian University, 1970.)
7. Klein, Paul A. *Toward a Theory of Sequencing: Study 1-1: An Exploration of Selected Relationships Among the Enactive, Iconic, and Symbolic Modes of Representation*. (Unpublished doctoral dissertation, The Pennsylvania State University, 1970.)
8. Kotarbinski, Taddeus "Praxiological Sentences and How They are Proved." *10th International Congress of Philosophy*.

9. Maccia, Elizabeth Steiner *The Model in Theorizing and Research*. Occasional Paper 65-180, Ohio State University: Educational Theory Center, 1965.
10. Maccia, Elizabeth Steiner *Ways of Inquiry*. Occasional Paper 62-107, Ohio State University: Educational Theory Center, 1962.
11. Popper, Karl R. *The Logic of Scientific Discovery*. New York: Basic Books, 1961.
12. Sawada, Daiyo *Toward a Theory of Sequencing: Study 3-1: An Assessment of a Selected Set of Piagetian-derived Operators for the Generation of Effective Learning Hierarchies*. (Doctoral dissertation now underway at The Pennsylvania State University.)